1998 Calculus BC Free-Response Questions

6. A particle moves along the curve defined by the equation $y = x^3 - 3x$. The x–coordinate of the particle, $x(t)$, satisfies the equation $\dfrac{dx}{dt} = \dfrac{1}{\sqrt{2t+1}}$, for $t \geq 0$ with initial condition $x(0) = -4$.

   (a) Find $x(t)$ in terms of t.

   (b) Find $\dfrac{dy}{dt}$ in terms of t.

   (c) Find the location and speed of the particle at time $t = 4$.

1999

The College Board

Advanced Placement Examination

CALCULUS BC

SECTION II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.

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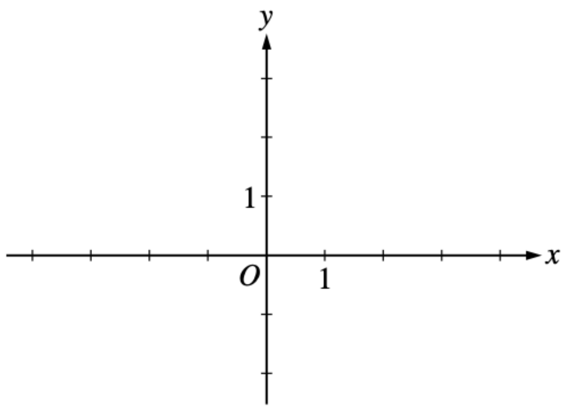
1. A particle moves in the xy-plane so that its position at any time t, $0 \le t \le \pi$, is given by $x(t) = \frac{t^2}{2} - \ln(1 + t)$ and $y(t) = 3 \sin t$.

   (a) Sketch the path of the particle in the xy-plane below. Indicate the direction of motion along the path.



   (b) At what time t, $0 \le t \le \pi$, does $x(t)$ attain its minimum value? What is the position $(x(t),\ y(t))$ of the particle at this time?

   (c) At what time t, $0 < t < \pi$, is the particle on the y-axis? Find the speed and the acceleration vector of the particle at this time.

2000 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

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4. A moving particle has position $(x(t),\ y(t))$ at time t. The position of the particle at time $t = 1$ is $(2, 6)$, and the velocity vector at any time $t > 0$ is given by $\left(1 - \frac{1}{t^2}, 2 + \frac{1}{t^2}\right)$.

   (a) Find the acceleration vector at time $t = 3$.

   (b) Find the position of the particle at time $t = 3$.

   (c) For what time $t > 0$ does the line tangent to the path of the particle at $(x(t),\ y(t))$ have a slope of 8 ?

   (d) The particle approaches a line as $t \to \infty$. Find the slope of this line. Show the work that leads to your conclusion.

2001 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with

$$\frac{dx}{dt} = \cos\left(t^3\right) \text{ and } \frac{dy}{dt} = 3\sin\left(t^2\right)$$

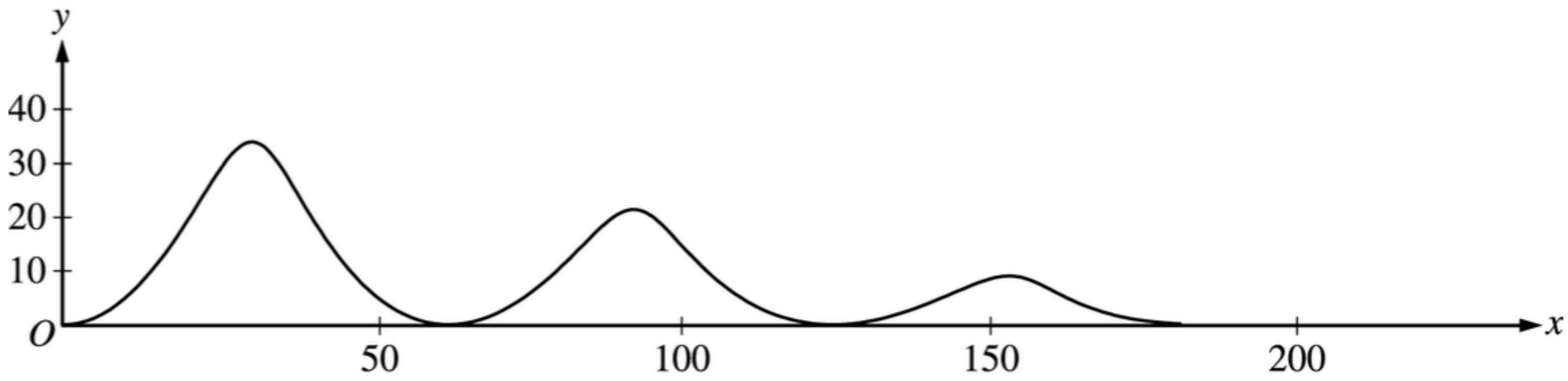
for $0 \le t \le 3$. At time $t = 2$, the object is at position $(4, 5)$.

(a) Write an equation for the line tangent to the curve at $(4, 5)$.

(b) Find the speed of the object at time $t = 2$.

(c) Find the total distance traveled by the object over the time interval $0 \le t \le 1$.

(d) Find the position of the object at time $t = 3$.



3. The figure above shows the path traveled by a roller coaster car over the time interval $0 \le t \le 18$ seconds. The position of the car at time t seconds can be modeled parametrically by

$$x(t) = 10t + 4\sin t$$
$$y(t) = (20 - t)(1 - \cos t),$$

where x and y are measured in meters. The derivatives of these functions are given by

$$x'(t) = 10 + 4\cos t$$
$$y'(t) = (20 - t)\sin t + \cos t - 1.$$

(a) Find the slope of the path at time $t = 2$. Show the computations that lead to your answer.

(b) Find the acceleration vector of the car at the time when the car's horizontal position is $x = 140$.

(c) Find the time t at which the car is at its maximum height, and find the speed, in m/sec, of the car at this time.

(d) For $0 < t < 18$, there are two times at which the car is at ground level $(y = 0)$. Find these two times and write an expression that gives the average speed, in m/sec, of the car between these two times. Do not evaluate the expression.

2002 AP® CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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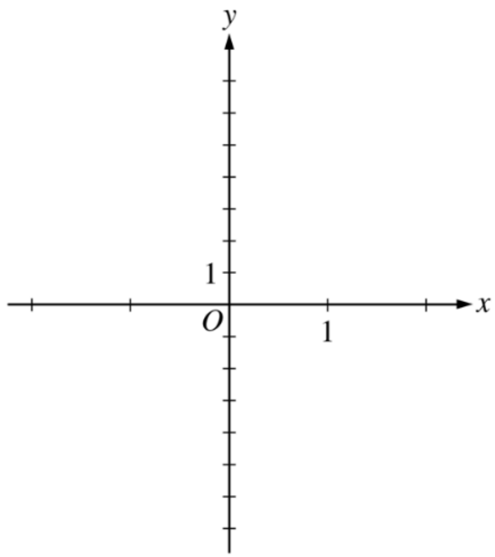
1. A particle moves in the xy-plane so that its position at any time t, for $-\pi \le t \le \pi$, is given by $x(t) = \sin(3t)$ and $y(t) = 2t$.

   (a) Sketch the path of the particle in the xy-plane provided. Indicate the direction of motion along the path. **(Note: Use the axes provided in the test booklet.)**



   (b) Find the range of $x(t)$ and the range of $y(t)$.

   (c) Find the smallest positive value of t for which the x-coordinate of the particle is a local maximum. What is the speed of the particle at this time?

   (d) Is the distance traveled by the particle from $t = -\pi$ to $t = \pi$ greater than 5π ? Justify your answer.

2003 AP® CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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4. A particle moves in the xy-plane so that the position of the particle at any time t is given by

$$x(t) = 2e^{3t} + e^{-7t} \text{ and } y(t) = 3e^{3t} - e^{-2t}.$$

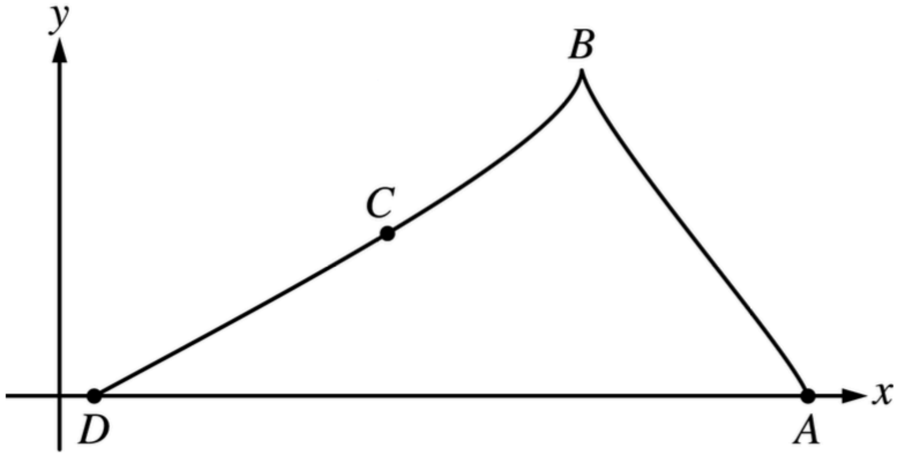
(a) Find the velocity vector for the particle in terms of t, and find the speed of the particle at time $t = 0$.

(b) Find $\frac{dy}{dx}$ in terms of t, and find $\lim_{t \to \infty} \frac{dy}{dx}$.

(c) Find each value t at which the line tangent to the path of the particle is horizontal, or explain why none exists.

(d) Find each value t at which the line tangent to the path of the particle is vertical, or explain why none exists.

2003 AP® CALCULUS BC FREE-RESPONSE QUESTIONS



2. A particle starts at point A on the positive x-axis at time $t = 0$ and travels along the curve from A to B to C to D, as shown above. The coordinates of the particle's position $(x(t), y(t))$ are differentiable functions of t, where $x'(t) = \frac{dx}{dt} = -9\cos\left(\frac{\pi t}{6}\right)\sin\left(\frac{\pi\sqrt{t+1}}{2}\right)$ and $y'(t) = \frac{dy}{dt}$ is not explicitly given. At time $t = 9$, the particle reaches its final position at point D on the positive x-axis.

(a) At point C, is $\frac{dy}{dt}$ positive? At point C, is $\frac{dx}{dt}$ positive? Give a reason for each answer.

(b) The slope of the curve is undefined at point B. At what time t is the particle at point B ?

(c) The line tangent to the curve at the point $(x(8), y(8))$ has equation $y = \frac{5}{9}x - 2$. Find the velocity vector and the speed of the particle at this point.

(d) How far apart are points A and D, the initial and final positions, respectively, of the particle?

2004 AP® CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. A particle moving along a curve in the plane has position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \sqrt{t^4 + 9} \text{ and } \frac{dy}{dt} = 2e^t + 5e^{-t}$$

for all real values of t. At time $t = 0$, the particle is at the point $(4, 1)$.

(a) Find the speed of the particle and its acceleration vector at time $t = 0$.

(b) Find an equation of the line tangent to the path of the particle at time $t = 0$.

(c) Find the total distance traveled by the particle over the time interval $0 \le t \le 3$.

(d) Find the x-coordinate of the position of the particle at time $t = 3$.

2004 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

3. An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with $\frac{dx}{dt} = 3 + \cos(t^2)$. The derivative $\frac{dy}{dt}$ is not explicitly given. At time $t = 2$, the object is at position $(1, 8)$.

   (a) Find the x-coordinate of the position of the object at time $t = 4$.

   (b) At time $t = 2$, the value of $\frac{dy}{dt}$ is -7. Write an equation for the line tangent to the curve at the point $(x(2), y(2))$.

   (c) Find the speed of the object at time $t = 2$.

   (d) For $t \geq 3$, the line tangent to the curve at $(x(t), y(t))$ has a slope of $2t + 1$. Find the acceleration vector of the object at time $t = 4$.

2005 AP® CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with

$$\frac{dx}{dt} = 12t - 3t^2 \text{ and } \frac{dy}{dt} = \ln\left(1 + (t-4)^4\right).$$

At time $t = 0,$ the object is at position $(-13, 5).$ At time $t = 2,$ the object is at point P with x-coordinate 3.

(a) Find the acceleration vector at time $t = 2$ and the speed at time $t = 2.$

(b) Find the y-coordinate of $P.$

(c) Write an equation for the line tangent to the curve at $P.$

(d) For what value of $t,$ if any, is the object at rest? Explain your reasoning.

2006 AP® CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

2. An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \tan\left(e^{-t}\right) \text{ and } \frac{dy}{dt} = \sec\left(e^{-t}\right)$$

for $t \geq 0$. At time $t = 1$, the object is at position $(2, -3)$.

(a) Write an equation for the line tangent to the curve at position $(2, -3)$.

(b) Find the acceleration vector and the speed of the object at time $t = 1$.

(c) Find the total distance traveled by the object over the time interval $1 \leq t \leq 2$.

(d) Is there a time $t \geq 0$ at which the object is on the y-axis? Explain why or why not.

3. An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \sin^{-1}\left(1 - 2e^{-t}\right) \text{ and } \frac{dy}{dt} = \frac{4t}{1 + t^3}$$

for $t \geq 0$. At time $t = 2$, the object is at the point $(6, -3)$. (Note: $\sin^{-1} x = \arcsin x$)

(a) Find the acceleration vector and the speed of the object at time $t = 2$.

(b) The curve has a vertical tangent line at one point. At what time t is the object at this point?

(c) Let $m(t)$ denote the slope of the line tangent to the curve at the point $(x(t), y(t))$. Write an expression for $m(t)$ in terms of t and use it to evaluate $\lim\limits_{t \to \infty} m(t)$.

(d) The graph of the curve has a horizontal asymptote $y = c$. Write, but do not evaluate, an expression involving an improper integral that represents this value c.

2007 AP® CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

2. An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t with

$$\frac{dx}{dt} = \arctan\left(\frac{t}{1+t}\right) \text{ and } \frac{dy}{dt} = \ln\left(t^2 + 1\right)$$

for $t \geq 0$. At time $t = 0$, the object is at position $(-3, -4)$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the speed of the object at time $t = 4$.

(b) Find the total distance traveled by the object over the time interval $0 \leq t \leq 4$.

(c) Find $x(4)$.

(d) For $t > 0$, there is a point on the curve where the line tangent to the curve has slope 2. At what time t is the object at this point? Find the acceleration vector at this point.

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. A particle moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with

$$\frac{dx}{dt} = \sqrt{3t} \text{ and } \frac{dy}{dt} = 3\cos\left(\frac{t^2}{2}\right).$$

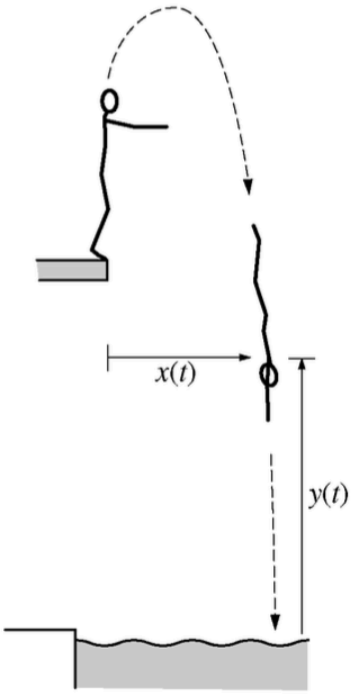
The particle is at position $(1, 5)$ at time $t = 4$.

(a) Find the acceleration vector at time $t = 4$.

(b) Find the y-coordinate of the position of the particle at time $t = 0$.

(c) On the interval $0 \leq t \leq 4$, at what time does the speed of the particle first reach 3.5 ?

(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 4$.



Note: Figure not drawn to scale.

3. A diver leaps from the edge of a diving platform into a pool below. The figure above shows the initial position of the diver and her position at a later time. At time t seconds after she leaps, the horizontal distance from the front edge of the platform to the diver's shoulders is given by $x(t)$, and the vertical distance from the water surface to her shoulders is given by $y(t)$, where $x(t)$ and $y(t)$ are measured in meters. Suppose that the diver's shoulders are 11.4 meters above the water when she makes her leap and that

$$\frac{dx}{dt} = 0.8 \text{ and } \frac{dy}{dt} = 3.6 - 9.8t,$$

for $0 \le t \le A$, where A is the time that the diver's shoulders enter the water.

(a) Find the maximum vertical distance from the water surface to the diver's shoulders.

(b) Find A, the time that the diver's shoulders enter the water.

(c) Find the total distance traveled by the diver's shoulders from the time she leaps from the platform until the time her shoulders enter the water.

(d) Find the angle θ, $0 < \theta < \frac{\pi}{2}$, between the path of the diver and the water at the instant the diver's shoulders enter the water.

2010 AP® CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

2. The velocity vector of a particle moving in the xy-plane has components given by

$$\frac{dx}{dt} = 14\cos\left(t^2\right)\sin\left(e^t\right) \text{ and } \frac{dy}{dt} = 1 + 2\sin\left(t^2\right), \text{ for } 0 \le t \le 1.5.$$

At time $t = 0$, the position of the particle is $(-2, 3)$.

(a) For $0 < t < 1.5$, find all values of t at which the line tangent to the path of the particle is vertical.

(b) Write an equation for the line tangent to the path of the particle at $t = 1$.

(c) Find the speed of the particle at $t = 1$.

(d) Find the acceleration vector of the particle at $t = 1$.

3. A particle is moving along a curve so that its position at time t is $(x(t), y(t))$, where $x(t) = t^2 - 4t + 8$ and $y(t)$ is not explicitly given. Both x and y are measured in meters, and t is measured in seconds. It is known that $\frac{dy}{dt} = te^{t-3} - 1$.

   (a) Find the speed of the particle at time $t = 3$ seconds.

   (b) Find the total distance traveled by the particle for $0 \le t \le 4$ seconds.

   (c) Find the time t, $0 \le t \le 4$, when the line tangent to the path of the particle is horizontal. Is the direction of motion of the particle toward the left or toward the right at that time? Give a reason for your answer.

   (d) There is a point with x-coordinate 5 through which the particle passes twice. Find each of the following.

      (i) The two values of t when that occurs

      (ii) The slopes of the lines tangent to the particle's path at that point

      (iii) The y-coordinate of that point, given $y(2) = 3 + \frac{1}{e}$

CALCULUS BC
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

1. At time t, a particle moving in the xy-plane is at position $(x(t), y(t))$, where $x(t)$ and $y(t)$ are not explicitly given. For $t \geq 0$, $\frac{dx}{dt} = 4t + 1$ and $\frac{dy}{dt} = \sin(t^2)$. At time $t = 0$, $x(0) = 0$ and $y(0) = -4$.

   (a) Find the speed of the particle at time $t = 3$, and find the acceleration vector of the particle at time $t = 3$.

   (b) Find the slope of the line tangent to the path of the particle at time $t = 3$.

   (c) Find the position of the particle at time $t = 3$.

   (d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

2. For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\dfrac{dx}{dt} = \dfrac{\sqrt{t+2}}{e^t}$ and $\dfrac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

(b) Find the x-coordinate of the particle's position at time $t = 4$.

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

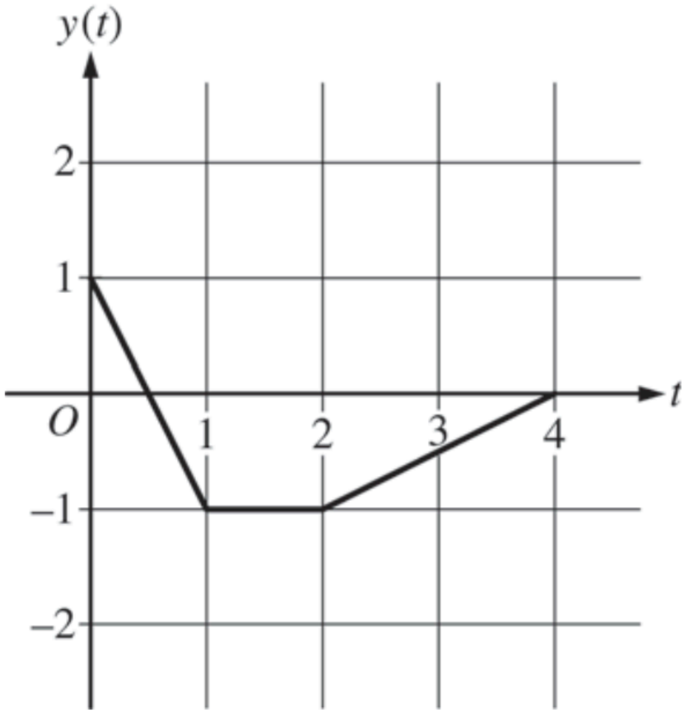
2. At time $t \geq 0$, a particle moving along a curve in the xy-plane has position $(x(t), y(t))$ with velocity vector $v(t) = \left(\cos\left(t^2\right), e^{0.5t}\right)$. At $t = 1$, the particle is at the point $(3, 5)$.

   (a) Find the x-coordinate of the position of the particle at time $t = 2$.

   (b) For $0 < t < 1$, there is a point on the curve at which the line tangent to the curve has a slope of 2. At what time is the object at that point?

   (c) Find the time at which the speed of the particle is 3.

   (d) Find the total distance traveled by the particle from time $t = 0$ to time $t = 1$.



2. At time t, the position of a particle moving in the xy-plane is given by the parametric functions $(x(t), y(t))$, where $\frac{dx}{dt} = t^2 + \sin(3t^2)$. The graph of y, consisting of three line segments, is shown in the figure above. At $t = 0$, the particle is at position $(5, 1)$.

(a) Find the position of the particle at $t = 3$.

(b) Find the slope of the line tangent to the path of the particle at $t = 3$.

(c) Find the speed of the particle at $t = 3$.

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

2. Researchers on a boat are investigating plankton cells in a sea. At a depth of h meters, the density of plankton cells, in millions of cells per cubic meter, is modeled by $p(h) = 0.2h^2e^{-0.0025h^2}$ for $0 \le h \le 30$ and is modeled by $f(h)$ for $h \ge 30$. The continuous function f is not explicitly given.

(a) Find $p'(25)$. Using correct units, interpret the meaning of $p'(25)$ in the context of the problem.

(b) Consider a vertical column of water in this sea with horizontal cross sections of constant area 3 square meters. To the nearest million, how many plankton cells are in this column of water between $h = 0$ and $h = 30$ meters?

(c) There is a function u such that $0 \le f(h) \le u(h)$ for all $h \ge 30$ and $\int_{30}^{\infty} u(h)\,dh = 105$. The column of water in part (b) is K meters deep, where $K > 30$. Write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column. Explain why the number of plankton cells in the column is less than or equal to 2000 million.

(d) The boat is moving on the surface of the sea. At time $t \ge 0$, the position of the boat is $(x(t), y(t))$, where $x'(t) = 662\sin(5t)$ and $y'(t) = 880\cos(6t)$. Time t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Find the total distance traveled by the boat over the time interval $0 \le t \le 1$.